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Complexity Analysis of Physiological Time Series Using a Novel Permutation-Ratio Entropy

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ABSTRACT To date, various information entropy methods have been employed to evaluate complexity within physiological time series. However, such methods cannot discern different levels of nonlinear chaotic properties within time series, indicating that incorrect results are yielded due to noise. Herein, a novel permutation-ratio entropy (PRE) method was proposed and compared with the classical permutation entropy (PE) method, multiscale PE with scale factors 4 and 8 (MPE_S4 and MPE_S8). Simulations with clean logistic mapping series and the logistic mapping series plus noise with a signal-to-noise ratio of 20 dB showed that only PRE monotonically declined with complexity reduction within time series for all 12 combinations of parameters (time delay τ and embedded dimension m). By contrast, PE only monotonically decreased at three parameter combinations for the clean logistic series and failed at all 12 parameter combinations for the logistic series plus noise, and moreover, MPE_S4 and MPE_S8 failed to monotonically decline for the clean logistic series and the logistic series plus noise at all parameter combinations. Results of surrogate data analysis indicated that PRE could more effectively measure the deterministic components of nonlinear within time series than PE, MPE_S4 and MPE_S8. In addition, the parameter m could enable PE, MPE_S4, and MPE_S8 to yield incorrect results, but it could not do so for PRE. Both PRE and PE were relatively stable on various parameters of τ. Interictal and ictal electroencephalography (EEG) recordings from the Bonn database and the CHB-MIT scalp EEG database were also observed, and the results indicated that the PRE could accurately measure the complexity of EEG recordings, as shown by higher entropy values yielded from interictal intracranial EEG recordings versus those yielded from ictal ones ($p < 0.01$).

INDEX TERMS Permutation-ratio entropy (pre), complexity, chaotic properties, physiological time series.

I. INTRODUCTION

Permutation entropy (PE) has been widely used to analyze complexity within time series because of its favorable performance [1]–[3]. Yan *et al.* [2] employed PE to characterize working status of rotary machines. PE could provide satisfactory results when detecting weak abrupt information hidden within time series [3]. In biomedical applications, Cao *et al.* [4] employed PE to identify different phases of epileptic activity in the intracranial EEG signals recorded from three epileptic patients. Nicolaou and Georgiou [5] investigated the use of PE as a feature for automated epileptic seizure detection. The study of Li *et al.* [6] demonstrated PE can be used not only to track the dynamical changes of EEG data, but also to successfully detect pre-seizure states. Veisi *et al.* [7] used PE to classify normal and epileptic EEG, and classification result for clean EEG recordings was up to 97% and more than that for highly noisy EEG recordings. Generally, physiological signals are weak nonlinear time series, and they are usually contaminated by noise and other random components. Therefore, it is difficult to accurately measure the inherent nonlinear complexity within a given physiological signal. The PE method can be used to measure the complexity of physiological series because it is more sensitive to abrupt change of signal and dynamic changes compared with other information entropy methods [3], [4]. However the PE method is performed on a symbol series instead of an original time series so that it loses details within series, and the method is based on sorting among data points of time series, which means that it neglects different degrees of amplitude among data points. Consequently, the PE method loses some information regarding the amplitude of time series and can provide incorrect results when measuring the complexity of physiological signals. Fig. 1 shows loss of amplitude information of PE so that three different series are sorted as the same series by PE.

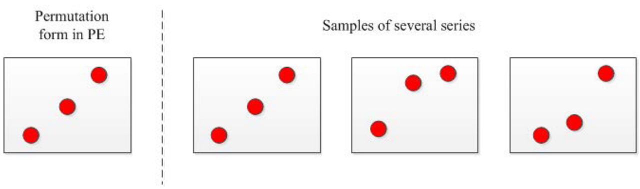

FIGURE 1. **Amplitude information loss of PE.**

Therefore, variants of PE have been designed to further improve performance. Bian *et al.* [8] proposed a modified PE method to map equal values onto the same symbols (ranks), allowing for a more accurate characterization of system states. Another common variant is multiscale PE (MPE), which uses various scale factors to reflect the detailed information within time series [9]–[11]. Liu *et al.* [9] employed MPE to analyze electrocardiogram signals. Li *et al.* [10] used MPE to analyze electroencephalography (EEG) recordings during sevoflurane anesthesia. The modified PE method solves only a greater number of equal values existed in the observed time series. However, it cannot yet reflect the amplitude difference of time series. The MPE exhibits only the trends of entropy values across time scales. Yi and Shang [12] and Fadlallah *et al.* [13] calculated weights by applying the variance of each neighbor vector to reflect the amplitude difference of time series, and this method could achieve satisfactory results on financial time series. However, the weighted PE method is more suited to stronger signals with considerable amplitude information [12]. Actual physiological signals are weak time series collected from the body that can easily be contaminated by noise and body motion, so the inherent amplitude of physiological signals is affected by random components. Thus, the weighted PE may not be suitable for use with physiological signals.

In this study, a novel permutation-ratio entropy (PRE) method was proposed for evaluating the complexity of time series. This method could reflect difference of amplitude between two adjacent data points in a time series. The performance of the PRE method was compared with the classical PE, the MPE with scale factor 4 (MPE_S4) and the MPE with scale factor 8 (MPE_S8) for both artificial time series and real EEG recordings obtained from the open Bonn database and the CHB-MIT scalp EEG database.

II. METHOD AND MATERIALS

A. THE CLASSICAL PE

The computation of PE is not relatively complexity. A given time series is transformed into a symbol series with relatively few symbols. After the symbolization it is possible to construct symbol series by collecting groups of symbols together in temporal order [14]. The detailed computation process of PE as follows.

According to the reconstruction theory, a time series $x(i), i = 1, 2, \ldots, n$, can be reconstructed as Eq. (1), as shown at the bottom of this page, where m is the embedded dimension and τ is the time delay.

Each row $X(i)$ of the reconstruction matrix X can be considered as a reconstructive component. Thus, the total number of reconstructive components is $n-(m-1)\tau$. It can be seen that for a given value of i, the m real values $X(i)$ are associated with numbers from 1 to m, and each $X(i)$ can be arranged in an increasing order as [14]

$$\begin{aligned} x(i+(j_1-1)\tau)) &\leq x(i+(j_2-1)\tau)) \\ &\leq \cdots x(i+(j_m-1)\tau)) \end{aligned} \tag{2}$$

where $j_1, j_2, \ldots, j_m$ represent the original position of each element in $X(i)$. If two or more elements in $X(i)$ that have the same value (e.g., $x(i+(j_1-1)\tau) = x(i+(j_2-1)\tau))$ can be sorted according to their original positions j_1 and j_2, then $x(i+(j_1-1)\tau) \leq x(i+(j_2-1)\tau)$ when $j_1 \leq j_2$. After the aforementioned sorting process, the matrix X can be described as a two-dimensional form as Eq. (3), as shown at the top of the next page.

Then, each sorted $X(i)$ is transformed into a sequence of integers $j1, j2, \ldots, jm$. Thus, each sorted $X(i)$ is associated with a permutation of the number sequence $1, 2, \ldots, m$. Disparate permutations represent different patterns. Being $m!$ the maximum number of distinct patterns capable of representing the analyzed time series, the probability distribution for these distinct patterns is $p_1, p_2, \ldots, p_k$ where $k \leq m!$. According to the Shannon entropy, the PE for the time series is defined as:

$$Hp = -\sum_{j=1}^{k} P_j \ln(P_j) \tag{4}$$

$$\begin{bmatrix} X(1) \\ \vdots \\ X(i) \\ \vdots \\ X(n-(m-1)\tau) \end{bmatrix} = \begin{bmatrix} x(1) & x(1+\tau) & \cdots & x(1+(m-1)\tau) \\ \vdots & \vdots & \cdots & \vdots \\ x(i) & x(i+\tau) & \cdots & x(i+(m-1)\tau) \\ \vdots & \vdots & \cdots & \vdots \\ x(n-(m-1)\tau) & x(n-(m-2)\tau) & \cdots & x(n) \end{bmatrix} \tag{1}$$

$$X_{n-(m-1)\tau,m} = \begin{bmatrix} X(1,1) & X(1,2) & \cdots & X(1,m) \\ \vdots & \vdots & \cdots & \vdots \\ X(i,1) & X(i,2) & \cdots & X(2,m) \\ \vdots & \vdots & \cdots & \vdots \\ X(n-(m-1)\tau,1) & X(n-(m-2)\tau,2) & \cdots & X(n-(m-1)\tau,m)) \end{bmatrix} \tag{3}$$

Finally a normalized PE is given as

$$PE = \frac{Hp}{\ln m!} \tag{5}$$

The classical PE algorithm has been detailed in [1] and [15].

B. MPE

MPE has to perform multiple successive coarse-grained processes for time series before calculating PE of the series. For a time series $x(i)$ of length n, multiple successive coarse-grained processes are performed by averaging the time data points within non-overlapping windows of increasing length s, which is called scale factor. Each element of the coarse-grained time series y_j^s is calculated as follows

$$y_j^s == \frac{1}{s}\sum_{i=(j-1)s+1}^{js} x_i,\ 1 \le j \le \frac{n}{s} \tag{6}$$

where n/s is the length of each coarse-grained time series. After multiple successive coarse-grained processes, the PE for each coarse-grained time series is calculated. The detailed MPE algorithm has been described in [9]–[11].

In this study, scale factors 4 and 8 were selected to calculate MPE respectively.

C. DEFINITION OF PRE

A new relation matrix B is constructed and each element of $B(i,j)$ is defined as

$$\begin{aligned} B(i,j) &= [X(i,j)/X(i,j-1)] \\ 1 &< i < n-m-1, 2 < j < m \end{aligned} \tag{7}$$

where $B(i,1) = 0$. Hence, the matrix B represents a relationship between adjacent elements.

The number of new patterns c must be calculated after the relation matrix B is achieved. Let $B(i)$ be the ith row vector of matrix B, and $c(i)$ be the number of the ith pattern, $1 \le i \le n-m-1$, and the initial of $c(i)$ is 1. The calculation process of c is detailed as follows.

For each $B(i)$, its corresponding $c(i)$ increases 1 when another vector $B(j)$, $(i \neq j)$ of matrix B is considered the same pattern as $B(i)$, that is, $B(i)$ is equal to $B(j)$ or their Pearson correlation coefficient r is ≤ -0.9 or ≥ 0.9, then the $B(j)$ is removed from the matrix B to avoid over counting while $c(j)$ is set to 0. It can be seen that the maximum total number of patterns c is $n-m-1$ when each vector of the matrix B represents new pattern. Finally, the total number of patterns c contained in the matrix B can be obtained. In this process, the correlation coefficient r reflects levels of similarity between patterns. According to rule of thumb for interpreting the strength of the correlation [16], [17], two patterns own very high correlation when the absolute value of their r is larger than 0.9.

Then, the probability P_i of the pattern $c(i)$ is calculated as

$$P_i = c(i)/\sum_{i=1}^{k} c(i) \tag{8}$$

where k is the total number of patterns c, $1 \le k \le n-m-1$.

According to the Shannon entropy, PRE is defined as follows

$$pre = -\sum_{j=1}^{k} P_j ln(P_j) \tag{9}$$

The total number of patterns achieves the maximum $n-m-1$ when each row $B(i)$ in the matrix B represents a new pattern, thus, the maximum PRE is

$$pre_{max} = -\sum_{j=1}^{n-m-1} \frac{1}{n-m-1} ln(\frac{1}{n-m-1}) \tag{10}$$

Lastly, the normalized PRE can be yielded as follows:

$$PRE = pre/(pre_{max}) \tag{11}$$

D. SIMULATED DATA

In this study, various artificial sequences, including Gaussian noise, logistic (Logi) mapping series and MIX(k) time series were used to observe the performance of PRE method. Gaussian noise and MIX(k) series were used to demonstrate the sensitivity of PRE to its parameters m and τ, whereas Logi series were employed to measure the monotonicity of PRE when complexity level μ of the models varied. Gaussian noise was yielded by the function *wgn* in MATLAB. The Logi series is given by

$$x_{n+1} = \mu \times x_n \times (1 - x_n),\ 1 < \mu \le 4 \tag{12}$$

where μ is an adjustable parameter, and the corresponding Logi mapping series (i.e., Logi3.6, Logi3.7, Logi3.8, Logi3.9, and Logi4.0) are identified as chaotic series when the values of μ are equal to 3.6, 3.7, 3.8, 3.9, and 4.0, respectively. The Logi mapping series is regarded as a periodic series when $\mu = 3.5$. The MIX(k) series of N points, where the range of k is between 0 and 1, is a Logi3.5 where $N \times k$ randomly chosen points have been replaced by random noise.

In this study, 20 samples were generated for each of the aforementioned artificial time series.

E. SENSITIVITY ANALYSIS OF PRE PARAMETERS

The performance of complexity measures may be affected by parameters of the measures. Thus, the sensitivity analysis of PRE to the parameters must be conducted.

The first test analyzed the effect of embedding dimension m and time delay τ on PRE and consisted of two schemes. The first scheme was proposed with values of τ varying from 1 to 12 in steps of one while m was equal to 6, and the second scheme was performed with m varying from 3 to 12 in steps of one while τ was equal to 6. In this test, the lengths of all time series were assigned to 2000.

F. MONOTONICITY ANALYSIS OF PRE

The objective of this test was to determine if PRE exhibited monotonicity with increasing chaotic properties within time series. For this purpose, the Logi mapping series were employed with different chaotic levels (i.e., Logi4.0, Logi3.9, Logi3.8, Logi3.7, Logi3.6, and Logi3.5) with a length of 2000.

G. EFFECTS OF NOISE ON PRE

In practical applications, physiological signals are usually contaminated by noise, which means it is difficult to distinguish system complexity. Hence, the signal complexity measured may be ambiguous even reversed at some complexity levels in noisy environments [18]. This test was designed to evaluate the complexity of the clean Logi mapping series plus noise (i.e., the clean Logi3.5, Logi3.6, Logi3.7, Logi3.8, Logi3.9, and Logi4.0 plus noise). In this test, Gaussian noise was added to the clear Logi mapping series on a predetermined signal-noise ratio (SNR), and the SNR was defined as follows:

$$SNR = 10 \times log_{10}(\frac{P_s}{P_n}) \tag{13}$$

where P_s and P_n denote the power of the clean signal and power of the noise, respectively. In most cases, the main waveform of physiological series cannot be identified when SNR $\leq$ 10 dB, indicating that the series cannot be used by clinical purpose. Hence, the SNR was set to 20 dB in this test.

In this study, surrogate data analysis was also used to further validate effects of noise on the PRE, PE, MPE_S4 and MPE_S8 methods. Initially, the technique basically of surrogate data analysis needs to specify a linear process as a null hypothesis, then several surrogate data sets according to the null hypothesis are generated. Finally a discriminating statistic is calculated for the original time series and all surrogate sets. If the values computed from the surrogate data are significantly different from those computed from the original data, then the null hypothesis is rejected and nonlinearity is detected [19]. In this study, the null hypothesis was that the surrogate data were consistent with the mean and variance of the original time series, and this hypothesis was generated through the linear correlation of Gaussian process. According to the null hypothesis, 20 surrogates for each realization of the logistic mapping (Logi4.0) process were initially generated using the Fourier transform algorithm [19]. The surrogate data could contaminate the complex structures in the logistic mapping process and increase the irregularity of the time series.

H. ANALYSIS OF REAL PHYSIOLOGICAL SERIES

In this study, two kinds of real epileptic EEG data were used to validate performance of PRE, and one was a real intracranial epileptic EEG data, the other was a scalp epileptic EEG data. In fact the intracranial epileptic EEG data were relatively clean because the data were recorded directly in the surface of brain, whereas the scalp epileptic EEG data were relatively easy to be contaminated so that the data contained more noise than the intracranial EEG data. The real intracranial epileptic EEG data were obtained from the Bonn database which was freely available online [20]–[22]. The data consisted of five datasets: A, B, C, D, and E. Sets A and B were recorded from five healthy subjects. Sets C, D and E consisted of EEG recordings collected from five patients with epilepsy, of which sets C and D consisted of interictal recordings, whereas set E contained only ictal recordings from epilepsy patients. Each dataset contained 100 recordings with 23.6 seconds in duration. All recordings were sampled at 173.61 Hz, which means that the length of each recording was approximately 4097 points. In this study, sets D and E were used to evaluate the performance of PRE. Detailed information regarding the five datasets is available online [22].

The scalp EEG data were taken from the CHB-MIT EEG database. The scalp EEG datasets were collected from 23 subjects (5 males, ages 3-22, and 17 females, ages 1.5-19, and subject 23 was a new case) with sampling rate 256 Hz [23], [24]. All subjects were from epileptic patients of the Children's Hospital Boston, and the subjects were monitored for up to several days following withdrawal of anti-seizure medication in order to characterize their seizures. For the EEG recordings of each subject, the recorded frequencies and durations of the seizures were different. The start and stop times of each epileptic seizure could be obtained from the annotation files of the CHB-MIT EEG database, and the times were signed by a professional electroencephalographer through examination of the EEG data. In this study, we firstly excluded the channel that was too short or severely contaminated by random noise. So we chose 23 subjects and then selected channel 9 in each subject, and we could get the ictal EEG data on the basis of the start and stop times of each epileptic seizure. Finally we generated 920 ictal recordings and interictal recordings with length of 4000 respectively.

We also calculated the classical PE, MPE_S4 and MPE_S8 for the aforementioned simulations and the real EEG recordings to compare performance with PRE.

III. RESULTS

A. RESULTS OF EFFECTS OF PARAMETERS

Fig. 2 shows the change trend of PRE, PE, MPE_S4 and MPE_S8 for Gaussian noise, MIX40, and MIX10 with time delay τ from 1 to 12 when the parameter $m = 6$. As illustrated in Fig. 2, both PRE and PE remained relatively stable on all of τ for the three aforementioned artificial series. MPE_S4 decreased with increasing of τ for MIX40 and MIX10, and MPE_S8 was observed to descend for all three artificial series with increasing of τ. For PRE, PE and MPE_S4, Gaussian noise had the highest complexity values at all τ, followed by the MIX40 and MIX10 series. MPE_S8 values of Gaussian noise overlapped with that of the MIX40, and MPE_S8 for the MIX10 yielded the minimum values.

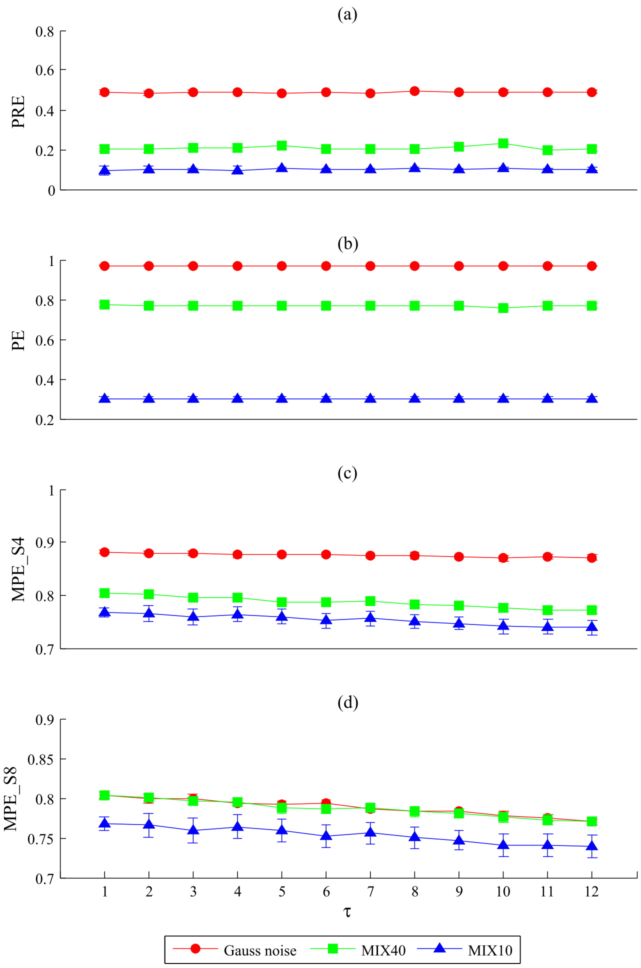

FIGURE 2. Change curves of PRE, PE, MPE_S4 and MPE_S8 for artificial time series with τ varying from 1 to 12 when $m = 6$. (a) PRE, (b) PE, (c) MPE_S4 and (d) MPE_S8.

Fig. 3 illustrates the values of PRE, PE, MPE_S4 and MPE_S8 for the three artificial time series (i.e., for Gaussian noise, MIX40 and MIX10 with various values of m from 3 to 12 when $\tau = 6$). As shown in Fig. 3, PRE, PE and MPE_S4 for Gaussian noise achieved higher values than that for the MIX40 and MIX10, whereas MIX10 again yielded the lowest values for both PRE and PE. The values of PRE for Gaussian noise, with MIX40 and MIX10 increasing monotonically with increasing m from 3 to 12. However, the curves of PE, MPE_S4 and MPE_S8 for the three artificial time series decreased monotonically with increasing m. Notably, differences between PRE values for Gaussian noise and MIX40 on all of m increased with increasing m, but differences between PE values for Gaussian noise and MIX40 decreased with increasing m, to the extent that even PE overlapped when $m = 12$. The curves of MPE_S8 for three artificial time series overlapped when $m = 7, 8, 9, 10, 11$ and 12 respectively.

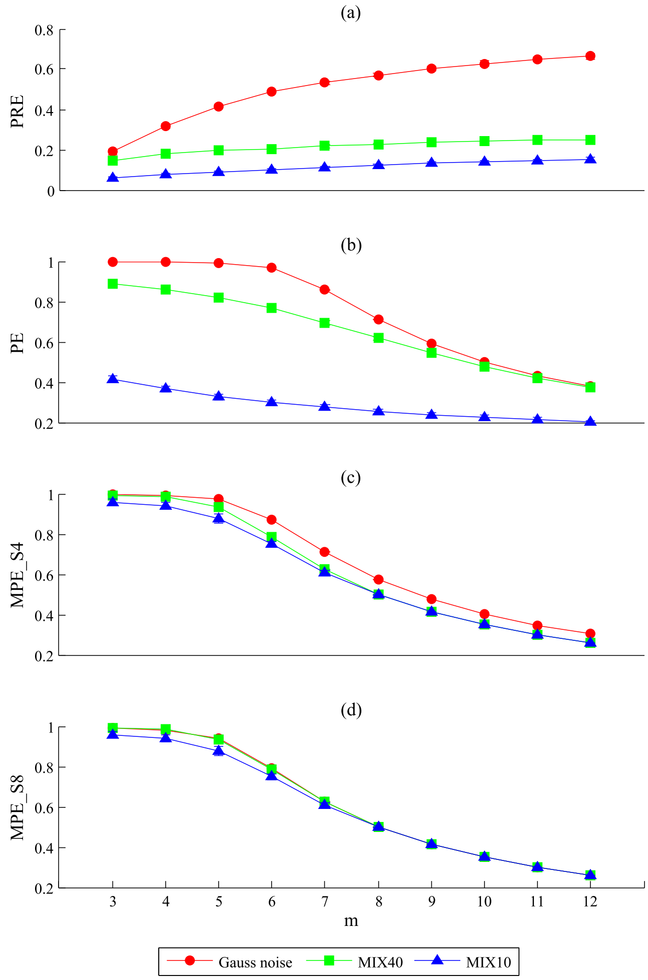

FIGURE 3. Change curves of PRE, PE, MPE_S4 and MPE_S8 for artificial time series with m varying from 3 to 12 when $\tau = 6$. (a) PRE, (b) PE, (c) MPE_S4 and (d) MPE_S8.

B. RESULTS OF MONOTONICITY ANALYSIS OF PRE

Fig. 4 shows PRE, PE, MPE_S4 and MPE_S8 for Logi4.0, Logi3.9, Logi3.8, Logi3.7, Logi3.6, and Logi3.5 with 12 combinations of the parameters τ and m (i.e., values of τ varying from 2, 6, and 10 and values of m varying from 3, 5, 7, and 9). PRE decreased monotonically in the order

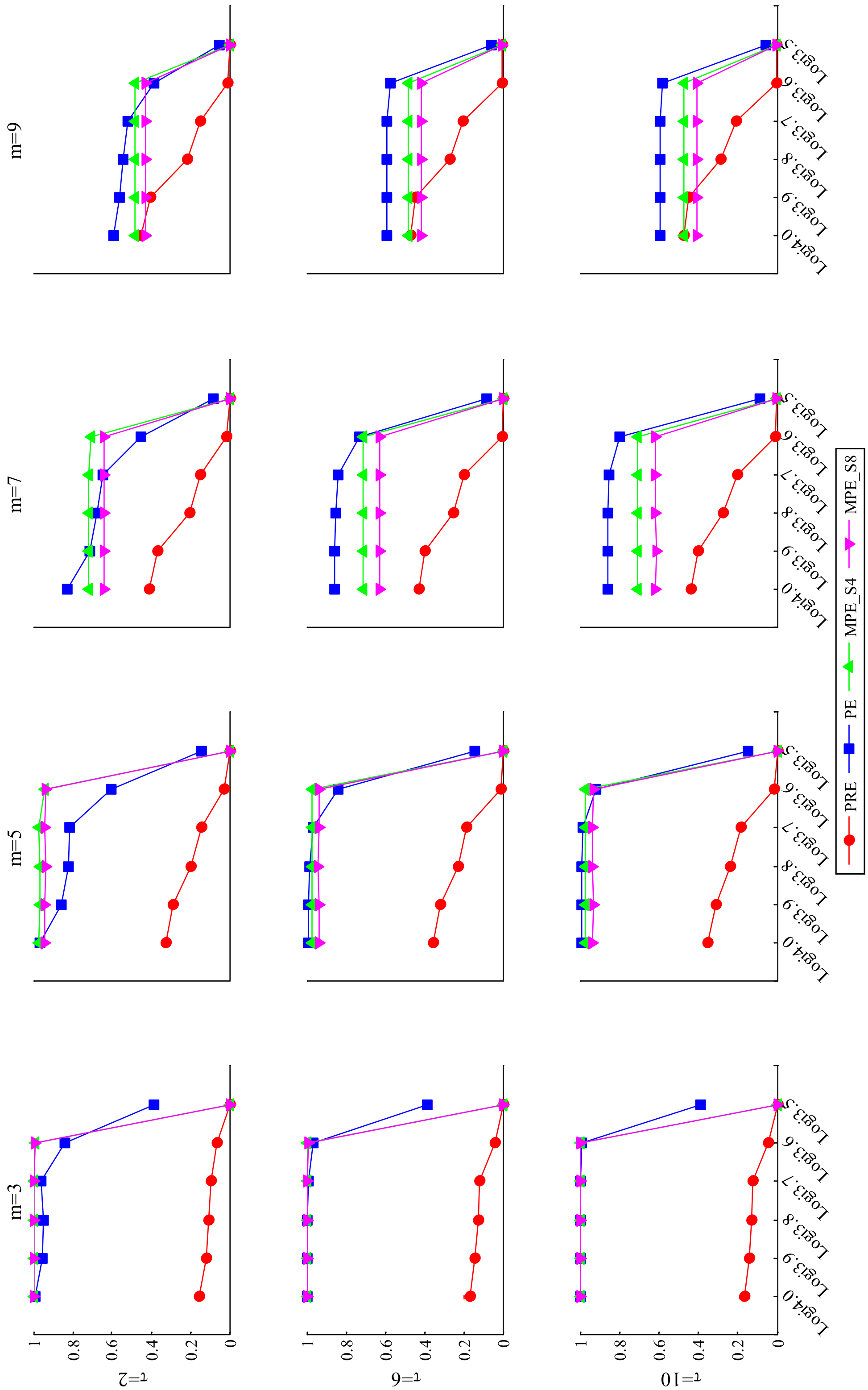

FIGURE 4. **Curves of PRE, PE, MPE_S4 and MPE_S8 on the clean logistic mapping series.**

Log4.0, Logi3.9, Logi3.8, Logi3.7, Logi3.6, and Logi3.5 for all 12 parameter pairs (τ, m). However, PE only decreased monotonically in the aforementioned order for two parameter pairs (i.e., 2, 7 and 2, 9). MPE_S4 and MPE_S8 yielded consistent values for Logi4.0, Logi3.9, Logi3.8, Logi3.7, and Logi3.6 at all parameter combinations.

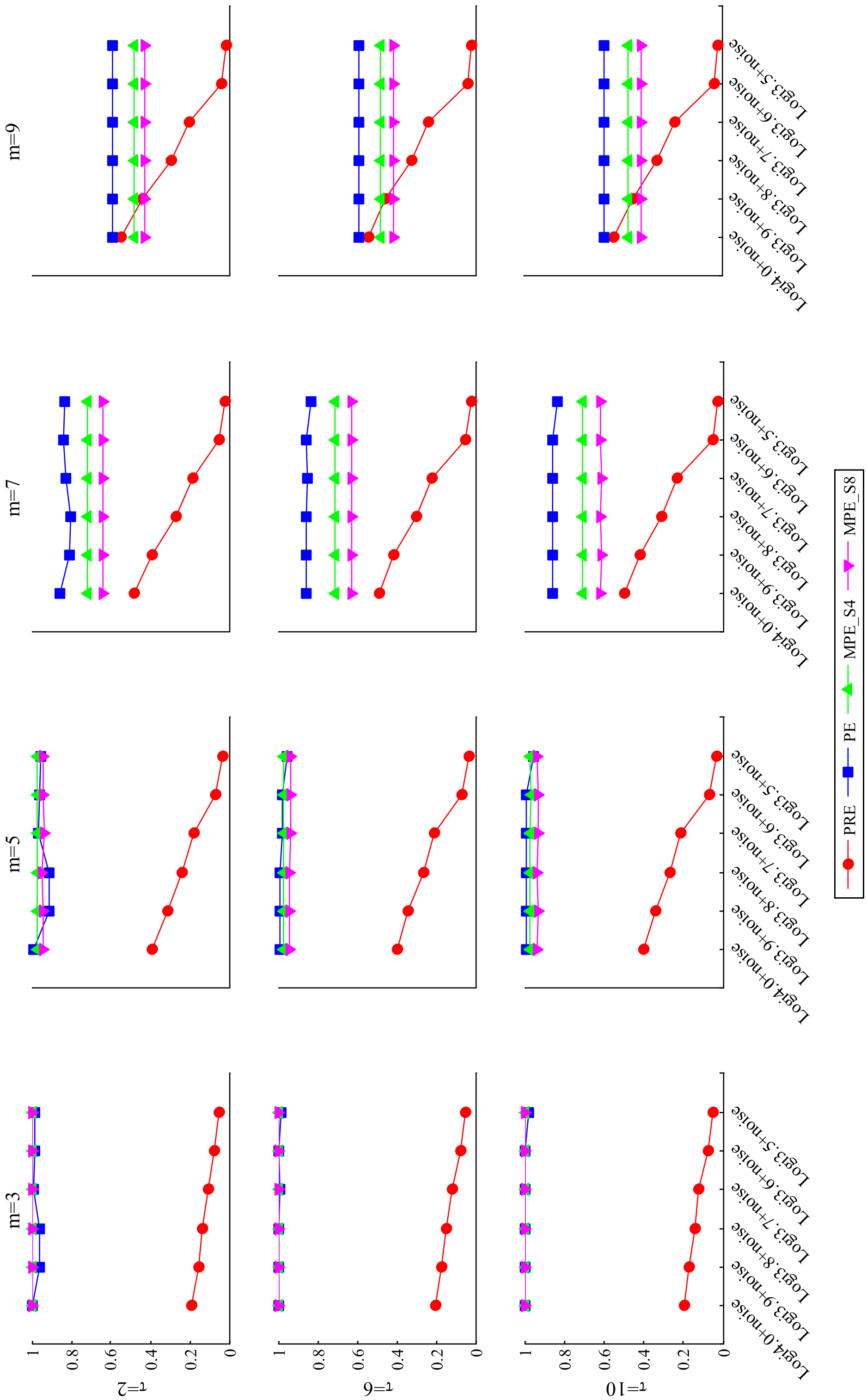

FIGURE 5. Curves of PRE, PE, MPE_S4 and MPE_S8 on the clean logistic mapping series plus Gaussian noise (SNR = 20 dB).

C. RESULTS OF NOISE EFFECT

Fig. 5 shows PRE, PE, MPE_S4 and MPE_S8 for noisy Logi mapping series (i.e., Logi4.0 plus Gaussian noise, Logi3.9 plus Gaussian noise, Logi3.8 plus Gaussian noise, Logi3.7 plus Gaussian noise, Logi3.6 plus Gaussian noise, and Logi3.5 plus Gaussian noise) with 12 combinations of the parameters τ and m (i.e., values of τ varying from 2, 6, and 10 and values of m varying from 3, 5, 7, and 9). As shown in Fig. 5, the values of PRE continue to decrease monotonically in the following order:

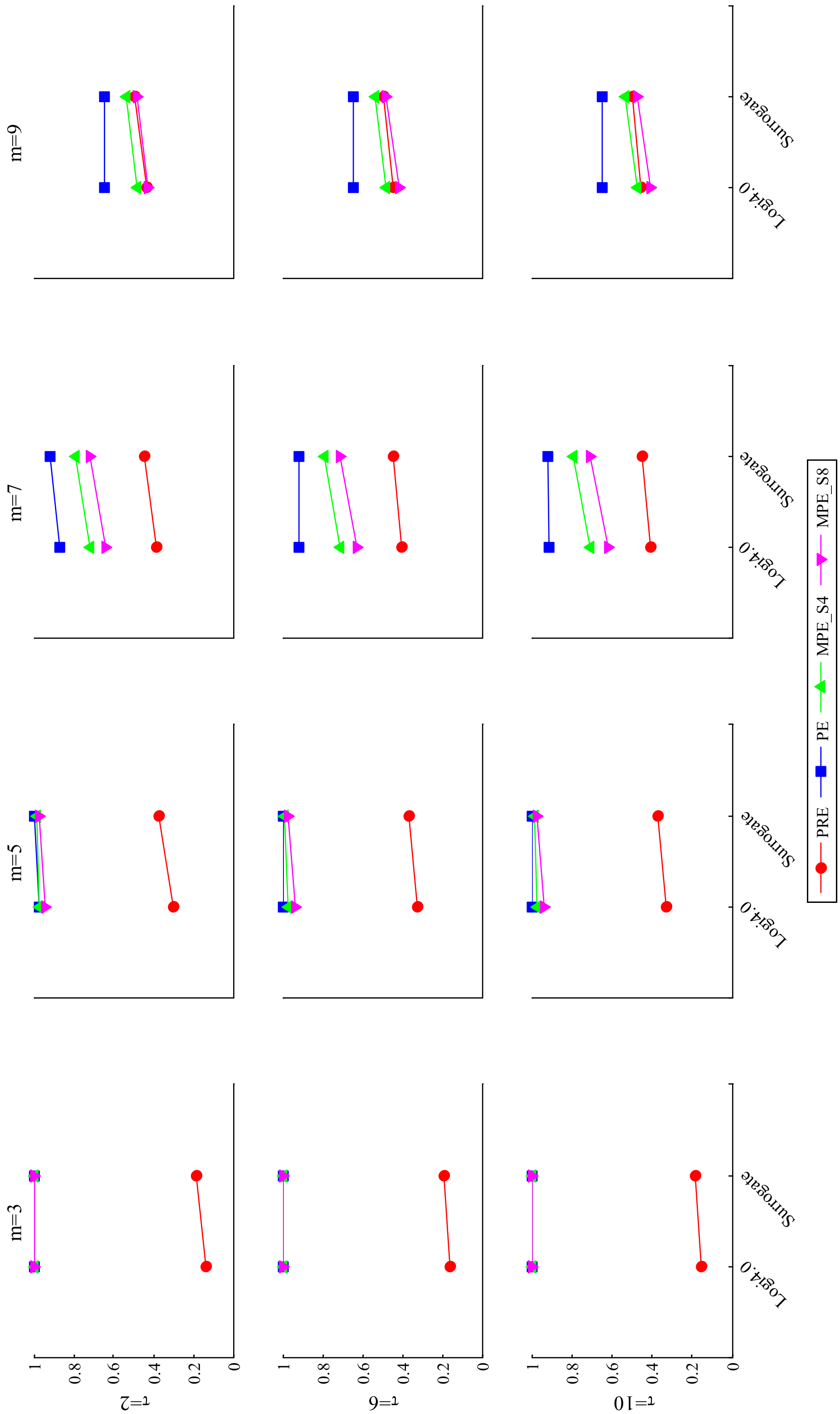

FIGURE 6. Curves of PRE, PE, MPE_S4 and MPE_S8 on the Logi4.0 and surrogate data.

Logi4.0 plus Gaussian noise, Logi3.9 plus Gaussian noise, Logi3.8 plus Gaussian noise, Logi3.7 plus Gaussian noise, Logi3.6 plus Gaussian noise, and Logi3.5 plus Gaussian noise with all 12 combinations of the parameters τ and m. However, PE did not decrease monotonically in the aforementioned order for all parameter combinations. MPE_S4 and

MPE_S8 kept stable for all noisy Logi mapping series at all parameter combinations.

Fig. 6 shows results of the surrogate data analysis. The values of PRE increased significantly for all 12 combinations of the parameters τ and m. However, PE did not demonstrate discernible increases when surrogate data was used for all parameter combinations except two combinations, i.e. (2, 5) and (2, 7). The values of MPE_S4 exhibited obviously increase between the Logi4.0 and the surrogate data for nine parameters combinations, i.e., (2, 5), (2, 7), (2, 9), (6, 7), (6, 9), (10, 7) and (10, 9), but MPE_S8 did not distinguished when $m = 3$.

D. RESULTS USING INTRACRANIAL EEG RECORDINGS

Fig. 7 presents the mean and standard deviation of PRE, PE, MPE_S4 and MPE_S8 values for both the real interictal and ictal groups taken from the Bonn database. The independent t test was performed to compare statistical differences in PRE, PE, MPE_S4 and MPE_S8 values between the interictal and ictal groups. Both PRE and PE had higher values for the interictal group than for the ictal group at all m. The differences in PRE values between the two groups were highly significant at every m (i.e., $m = 3$, 5, 7, and 9) because all p values were considerably lower than 0.01. The differences in terms of PE between the two groups were highly significant at $m = 7$ and 9. MPE_S4 between two groups exhibited significant difference at $m = 3$, 5 and 7. MPE_S8 values between the two groups had no significant difference at all m ($p > 0.05$).

E. RESULTS USING SCALP EEG RECORDINGS

Fig. 8 shows the mean and standard deviation of PRE, PE, MPE_S4 and MPE_S8 values for both the real interictal and ictal groups taken from the CHB-MIT EEG database. The PRE, PE and MPE_S4 methods yielded higher values for the interictal group than that for the ictal group. The PRE values between two groups were highly significant difference when $m = 5$, 7, and 9 ($p < 0.01$), and the PE values between two groups were highly significant when $m = 7$ ($p < 0.01$). For MPE_S4 and MPE_S8, highly significant differences were observed between two groups when $m = 5$.

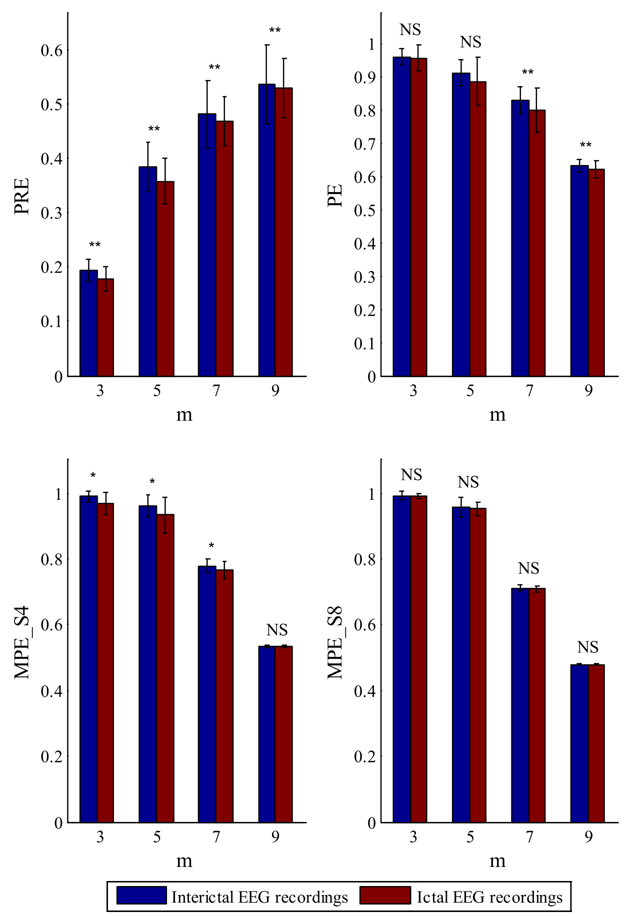

FIGURE 7. PRE, PE, MPE_S4 and MPE_S8 for the inter-ictal EEG recordings and ictal EEG recordings taken from the intracranial EEG recordings when $\tau = 6$, $m = 3, 5, 7, 9$. The p-value obtained from an independent samples t-test. (NS represents $p > 0.05$, * represents $p < 0.05$, and ** represents $p < 0.01$)

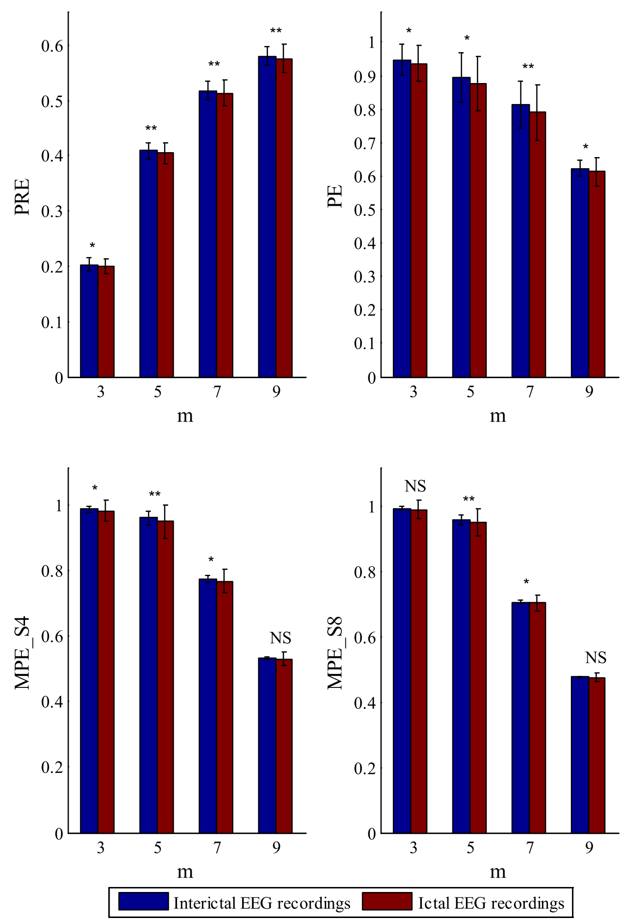

FIGURE 8. PRE, PE, MPE_S4 and MPE_S8 for the inter-ictal EEG recordings and ictal EEG recordings taken from the scalp EEG recordings when $\tau = 6$, $m = 3, 5, 7, 9$. The p-value obtained from an independent samples t-test. (NS represents $p > 0.05$, * represents $p < 0.05$, and ** represents $p < 0.01$)

IV. DISCUSSION

A satisfactory algorithm should be affected as little as possible as by its parameters. As illustrated in Fig. 2, PRE and PE were not sensitive to the parameter τ, because the two entropy methods could maintain relatively stable values when τ varied from 1 to 12. Moreover, PRE, PE and MPE_S4 could properly discern the complexity of Gaussian noise, MIX40, and MIX10 at all τ, because the values of the three entropy methods at each τ decreased in the order of Gaussian noise, MIX40, and MIX10. Fig. 3 shows that PRE is satisfactory in appropriately distinguishing the complexity of three artificial time series, because the entropy method at all m monotonically decrease in the order of Gaussian noise, MIX40, and MIX10. PE could properly discern three artificial series when the values of m were smaller. However, the performance of PE decreased when m was greater than 6, and PE could not discern the complexity of Gaussian noise and MIX40 when m was equal to 12, because the values of PE for Gaussian noise and MIX40 overlapped. According to Eq.(10) and (11), the embedded dimension m can affect the PRE method, and the values of PRE increase with the increasing of m. Similarly, PE is also affected by the parameter m according to Eq.(5), and the values of PE decrease with the increasing of m. As illustrated by Fig. 3, the theoretical analysis is in accordance with experimental results.

Furthermore, an important characteristic of PRE is that the algorithm can reflect amplitude change of time series so that PRE can more accurately calculate number of new patterns within time series than PE. In fact amplitude changes of Gaussian noise are much more than that of MIX40 because Gaussian noise is more irregular than MIX40. So the PRE values of Gaussian noise were higher than that of MIX40. New patterns within time series calculated by PE are not accurate because the method neglects many changes of amplitude of time series. As shown in Fig. 3, PE could not properly discern Gaussian noise and the MIX40 when the values of m were greater than 10. In fact the bigger m means a reconstructed vector is longer, and more information is contained in the vector, so the bigger m helps PRE to obtain much more new patterns within time series. As shown in Fig. 3, PRE could more cleanly distinguish Gaussian noise and the MIX40 when m was relatively larger. But for PE, the larger m yields the smaller PE value so that the PE values of Gaussian noise and that of the MIX40 cannot be discriminated because the PE values of the two time series are smaller and easy to overlap. Fig. 2 and Fig. 3 illustrated that PRE can maintain more favorable performance compared with the PE method. In contrast, MPE based on PE neglects more changes of amplitude of time series because the coarse-grained process of MPE loses essential data. As illustrated in Fig. 3, the greater scale factors s the MPE selected, the more apparent MPE could not distinguish Gaussian noise, the MIX40 and MIX10.

An effective algorithm should accurately differentiate complexity levels within signals rather than confusing them. As illustrated in Fig. 4, PRE could properly discern the complexity of Logi mapping series, because PRE decreased monotonically for 12 pairs of the parameter τ and m as follows: Logi4.0, Logi3.9, Logi3.8, Logi3.7, Logi3.6, and Logi3.5. By contrast, PE could only exhibit proper change trends when the combinations of the parameters τ and m were 2, 7; and 2, 9. MPE_S4 and MPE_S8 cannot properly discern at all parameter combinations. The results indicate that PRE has superior performance than PE for distinguishing chaotic properties within time series because it can more accurately calculated new patterns within time series but PE cannot, because PE ignores amplitude changes of time series. Both MPE_S4 and MPE_S8 failed to discern chaotic properties within time series because number of new patterns within time series were inaccurate since calculation of two algorithms needed to average time data points within non-overlapping windows so that more changes of time series were ignored.

Another common view is that a satisfactory algorithm should be able to accurately observe inherent properties within signals contaminated by noise. As shown in Fig. 5, the results of a noise effect demonstrated that PRE had satisfactory performance at all 12 combinations of the parameters τ and m for identifying chaotic properties within time series contaminated by noise, but PE, MPE_S4 and MPE_S8 failed. PRE is thus the most robust among PE, MPE_S4 and MPE_S8 in the contaminated time series. In fact, the three methods i.e., PE, MPE_S4 and MPE_S8 neglect not only noise but also chaotic properties within time series so that values of three methods keep constant or slight fluctuation for all of noisy Logi mapping series. Results of surrogate data analysis indicated PRE had a satisfactory performance for measuring nonlinear complexity within noisy time series. This is a critical aspect to consider while choosing a suitable measure for real field data applications.

Regarding the EEG data obtained from the Bonn database, the complexity of interictal EEG recordings from epilepsy patients is generally considered greater than that of ictal recordings [25]. In Fig. 7, PRE accurately measured the inherent complexity of intracranial EEG recordings by yielding greater PRE values for the interictal group. However, PE failed to provide accurate measurements. As shown in Fig. 8, PRE also exhibited a satisfactory performance for measuring complexity of the scalp epileptic EEG data. In general, PE lost a lot of detail information within time series including not only noise but also some inherent information so that PE could not exhibit a satisfactory performance to distinguish the interictal and ictal group. MPE_S4 and MPE_S8 were also similar to PE.

The performance of PRE is better than that of PE for various simulated data and real data because the method can provide more accurate number of patterns within time series than the classical PE. The PE loses detailed information within time series because each row of the reconstruction matrix needs to be mapped into a sequence of integers $j_1, j_2, \ldots, j_m$, so the maximum number of patterns within time series is $m!$.

For example, the number of all possible patterns is 6 when $m = 3$, however for the original time series instead of a sequence of integers, the number of patterns within series is larger than 6. For the PRE method, it runs on the original time series. Another important reason is that PRE can also reflect diverse amplitudes of time series to contribute to keep more details original time series, but the PE cannot.

V. CONCLUSION

This study proposed a novel PRE method that retains the amplitude information of nonlinear time series. The new entropy method can more accurately reflect whole information within time series than the classical PE, MPE_S4 and MPE_S8, especially in low SNR environments. PRE can accurately differentiate nonlinear chaotic levels with the logistic mapping sequence, and it can also correctly identify various logistic series plus noise. Lastly, the results using real EEG recordings from the Bonn database and the CHB-MIT EEG database confirmed that PRE's performance is superior to that of PE, MPE_S4 and MPE_S8.

Actually an in-depth discussion helps to explain the PRE results of the intracranial EEG data and the scalp EEG data. Therefore, our future work will mainly focus on finding a scientific and rational explanation in more extended ways.

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CONFLICT OF INTEREST STATEMENT

The authors declare that there are no conflicts of interest to this work.

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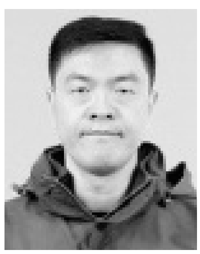

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